1. If s_1, s_2, s_3 are the sums of n, $2n$, $3n$ terms respectively of an arithmetical progression, shew that $s_3 = 3(s_2 - s_1)$.

2. Find two numbers such that their difference, sum and product, are to one another as 1, 7, 24.

3. In what scale of notation is 25 doubled by reversing the digits?

4. Solve the equations:

(1) $(x+2)(x+3)(x-4)(x-5) = 44$.

(2) $x(y+z)+2=0$, $y(z-2x)+21=0$, $z(2x-y)=5$.

5. In an A. P., of which a is the first term, if the sum of the first p terms $=0$, shew that the sum of the next q terms

$$= -\frac{a(p+q)q}{p-1}.$$

[R. M. A. WOOLWICH.]

6. Solve the equations:

(1) $(a+b)(ax+b)(a-bx) = (a^2x-b^2)(a+bx)$.

(2) $x^{\frac{1}{3}} + (2x-3)^{\frac{1}{3}} = \{12(x-1)\}^{\frac{1}{3}}$. [INDIA CIVIL SERVICE.]

7. Find an arithmetical progression whose first term is unity such that the second, tenth and thirty-fourth terms form a geometric series.

8. If α, β are the roots of $x^2+px+q=0$, find the values of

$$\alpha^2+\alpha\beta+\beta^2, \quad \alpha^3+\beta^3, \quad \alpha^4+\alpha^2\beta^2+\beta^4.$$

9. If $2x = a + a^{-1}$ and $2y = b + b^{-1}$, find the value of

$$xy + \sqrt{(x^2-1)(y^2-1)}.$$

10. Find the value of

$$\frac{(4+\sqrt{15})^{\frac{3}{2}} + (4-\sqrt{15})^{\frac{3}{2}}}{(6+\sqrt{35})^{\frac{3}{2}} - (6-\sqrt{35})^{\frac{3}{2}}}.$$

[R. M. A. WOOLWICH.]

11. If α and β are the imaginary cube roots of unity, shew that

$$\alpha^4+\beta^4+\alpha^{-1}\beta^{-1} = 0.$$

12. Shew that in any scale, whose radix is greater than 4, the number 12432 is divisible by 111 and also by 112.

13. A and B run a mile race. In the first heat A gives B a start of 11 yards and beats him by 57 seconds; in the second heat A gives B a start of 81 seconds and is beaten by 88 yards: in what time could each run a mile?

14. Eliminate x, y, z between the equations:

$$x^2-yz=a^2,\ y^2-zx=b^2,\ z^2-xy=c^2,\ x+y+z=0.$$

[R. M. A. Woolwich.]

15. Solve the equations:

$$ax^2+bxy+cy^2=bx^2+cxy+ay^2=d.$$

[Math. Tripos.]

16. A waterman rows to a place 48 miles distant and back in 14 hours: he finds that he can row 4 miles with the stream in the same time as 3 miles against the stream: find the rate of the stream.

17. Extract the square root of

(1) $(a^2+ab+bc+ca)(bc+ca+ab+b^2)(bc+ca+ab+c^2)$.

(2) $1-x+\sqrt{22x-15-8x^2}$.

18. Find the coefficient of x^6 in the expansion of $(1-3x)^{\frac{10}{3}}$, and the term independent of x in $\left(\frac{4}{3}x^2-\frac{3}{2x}\right)^9$.

19. Solve the equations:

(1) $\dfrac{2x-3}{x-1}-\dfrac{3x-8}{x-2}+\dfrac{x+3}{x-3}=0$.

(2) $x^2-y^2=xy-ab,\quad (x+y)(ax+by)=2ab(a+b)$.

[Trin. Coll. Camb.]

20. Shew that if $a(b-c)x^2+b(c-a)xy+c(a-b)y^2$ is a perfect square, the quantities a, b, c are in harmonical progression.

[St Cath. Coll. Camb.]

21. If

$$(y-z)^2+(z-x)^2+(x-y)^2=(y+z-2x)^2+(z+x-2y)^2+(x+y-2z)^2,$$

and x, y, z are real, shew that $x=y=z$. [St Cath. Coll. Camb.]

22. Extract the square root of 3e58261 in the scale of twelve, and find in what scale the fraction $\frac{1}{5}$ would be represented by ·17.

23. Find the sum of the products of the integers 1, 2, 3, ... n taken two at a time, and shew that it is equal to half the excess of the sum of the cubes of the given integers over the sum of their squares.

24. A man and his family consume 20 loaves of bread in a week. If his wages were raised 5 per cent., and the price of bread were raised $2\frac{1}{2}$ per cent., he would gain $6d.$ a week. But if his wages were lowered $7\frac{1}{2}$ per cent., and bread fell 10 per cent., then he would lose $1\frac{1}{2}d.$ a week: find his weekly wages and the price of a loaf.

25. The sum of four numbers in arithmetical progression is 48 and the product of the extremes is to the product of the means as 27 to 35: find the numbers.

26. Solve the equations:

(1) $a(b-c)x^2+b(c-a)x+c(a-b)=0.$

(2) $\dfrac{(x-a)(x-b)}{x-a-b}=\dfrac{(x-c)(x-d)}{x-c-d}.$ [MATH. TRIPOS.]

27. If $\sqrt{a-x}+\sqrt{b-x}+\sqrt{c-x}=0$, shew that

$$(a+b+c+3x)(a+b+c-x)=4(bc+ca+ab);$$

and if $\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}=0$, shew that $(a+b+c)^3=27abc.$

28. A train, an hour after starting, meets with an accident which detains it an hour, after which it proceeds at three-fifths of its former rate and arrives 3 hours after time: but had the accident happened 50 miles farther on the line, it would have arrived $1\frac{1}{2}$ hrs. sooner: find the length of the journey.

29. Solve the equations:

$$2x+y=2z,\quad 9z-7x=6y,\quad x^3+y^3+z^3=216.$$

[R. M. A. WOOLWICH.]

30. Six papers are set in examination, two of them in mathematics: in how many different orders can the papers be given, provided only that the two mathematical papers are not successive?

31. In how many ways can £5. 4s. 2d. be paid in exactly 60 coins, consisting of half-crowns, shillings and fourpenny-pieces?

32. Find a and b so that $x^3+ax^2+11x+6$ and $x^3+bx^2+14x+8$ may have a common factor of the form x^2+px+q.

[LONDON UNIVERSITY.]

33. In what time would A, B, C together do a work if A alone could do it in six hours more, B alone in one hour more, and C alone in twice the time?

34. If the equations $ax+by=1$, $cx^2+dy^2=1$ have only one solution prove that $\frac{a^2}{c}+\frac{b^2}{d}=1$, and $x=\frac{a}{c}$, $y=\frac{b}{d}$. [MATH. TRIPOS.]

35. Find by the Binomial Theorem the first five terms in the expansion of $(1-2x+2x^2)^{-\frac{1}{2}}$.

36. If one of the roots of $x^2+px+q=0$ is the square of the other, shew that $p^3-q(3p-1)+q^2=0$.

[PEMB. COLL. CAMB.]

37. Solve the equation

$$x^4-5x^2-6x-5=0.$$

[QUEEN'S COLL. OX.]

38. Find the value of a for which the fraction

$$\frac{x^3-ax^2+19x-a-4}{x^3-(a+1)x^2+23x-a-7}$$

admits of reduction. Reduce it to its lowest terms. [MATH. TRIPOS.]

39. If a, b, c, x, y, z are real quantities, and

$$(a+b+c)^2=3(bc+ca+ab-x^2-y^2-z^2),$$

shew that $a=b=c$, and $x=0$, $y=0$, $z=0$.

[CHRIST'S COLL. CAMB.]

40. What is the greatest term in the expansion of $\left(1-\frac{2}{3}x\right)^{-\frac{3}{5}}$ when the value of x is $\frac{6}{7}$? [EMM. COLL. CAMB.]

41. Find two numbers such that their sum multiplied by the sum of their squares is 5500, and their difference multiplied by the difference of their squares is 352. [CHRIST'S COLL. CAMB.]

42. If $x=\lambda a$, $y=(\lambda-1)b$, $z=(\lambda-3)c$, $\lambda=\frac{1+b^2+3c^2}{a^2+b^2+c^2}$, express $x^2+y^2+z^2$ in its simplest form in terms of a, b, c.

[SIDNEY COLL. CAMB.]

43. Solve the equations:

(1) $x^4+3x^2=16x+60$.

(2) $y^2+z^2-x=z^2+x^2-y=x^2+y^2-z=1$.

[CORPUS COLL. OX.]

44. If x, y, z are in harmonical progression, shew that

$$\log(x+z)+\log(x-2y+z)=2\log(x-z).$$

45. Shew that

$$\frac{1}{2}+\frac{1.3}{2.4}\left(\frac{1}{4}\right)+\frac{1.3.5}{2.4.6}\left(\frac{1}{4}\right)^2+\ldots\ldots=\frac{4}{3}(2-\sqrt{3})\sqrt{3}.$$

[EMM. COLL. CAMB.]

46. If $$\frac{3x+2y}{3a-2b}=\frac{3y+2z}{3b-2c}=\frac{3z+2x}{3c-2a},$$

then will $$5(x+y+z)(5c+4b-3a)=(9x+8y+13z)(a+b+c).$$

[CHRIST'S COLL. CAMB.]

47. With 17 consonants and 5 vowels, how many words of four letters can be formed having 2 different vowels in the middle and 1 consonant (repeated or different) at each end?

48. A question was lost on which 600 persons had voted; the same persons having voted again on the same question, it was carried by twice as many as it was before lost by, and the new majority vote was to the former as 8 to 7: how many changed their minds? [ST JOHN'S COLL. CAMB.]

49. Shew that

$$\log\frac{(1+x)^{\frac{1-x}{2}}}{(1-x)^{\frac{1+x}{2}}}=x+\frac{5x^3}{2.3}+\frac{9x^5}{4.5}+\frac{13x^7}{6.7}+\ldots$$

[CHRIST'S COLL. CAMB.]

50. A body of men were formed into a hollow square, three deep, when it was observed, that with the addition of 25 to their number a solid square might be formed, of which the number of men in each side would be greater by 22 than the square root of the number of men in each side of the hollow square: required the number of men.

51. Solve the equations:

(1) $\sqrt[m]{(a+x)^2}+2\sqrt[m]{(a-x)^2}=3\sqrt[m]{a^2-x^2}.$

(2) $(x-a)^{\frac{1}{2}}(x-b)^{\frac{1}{2}}-(x-c)^{\frac{1}{2}}(x-d)^{\frac{1}{2}}=(a-c)^{\frac{1}{2}}(b-d)^{\frac{1}{2}}.$

52. Prove that

$$\sqrt[3]{4}=1+\frac{2}{6}+\frac{2.5}{6.12}+\frac{2.5.8}{6.12.18}+\ldots\ldots$$

[SIDNEY COLL. CAMB.]

53. Solve $\sqrt[3]{6(5x+6)}-\sqrt[3]{5(6x-11)}=1.$

[QUEENS' COLL. CAMB.]

54. A vessel contains a gallons of wine, and another vessel contains b gallons of water: c gallons are taken out of each vessel and transferred to the other; this operation is repeated any number of times: shew that if $c(a+b)=ab$, the quantity of wine in each vessel will always remain the same after the first operation.

55. The arithmetic mean between m and n and the geometric mean between a and b are each equal to $\dfrac{ma+nb}{m+n}$: find m and n in terms of a and b.

56. If x, y, z are such that their sum is constant, and if

$$(z+x-2y)(x+y-2z)$$

varies as yz, prove that $2(y+z)-x$ varies as yz.

[EMM. COLL. CAMB.]

57. Prove that, if n is greater than 3,

$$1\,.\,2\,.\,{}^nC_r-2\,.\,3\,.\,{}^nC_{r-1}+3\,.\,4\,.\,{}^nC_{r-2}-\ldots\ldots+(-1)^r(r+1)(r+2)=2\,.\,{}^{n-3}C_r.$$

[CHRIST'S COLL. CAMB.]

58. Solve the equations:

(1) $\sqrt{2x-1}+\sqrt{3x-2}=\sqrt{4x-3}+\sqrt{5x-4}$.

(2) $4\{(x^2-16)^{\frac{3}{4}}+8\}=x^2+16(x^2-16)^{\frac{1}{4}}$.

[ST JOHN'S COLL. CAMB.]

59. Prove that two of the quantities x, y, z must be equal to one another, if

$$\frac{y-z}{1+yz}+\frac{z-x}{1+zx}+\frac{x-y}{1+xy}=0.$$

60. In a certain community consisting of p persons, a per cent. can read and write; of the males alone b per cent., and of the females alone c per cent. can read and write: find the number of males and females in the community.

61. If $x=\left(\dfrac{a}{b}\right)^{\frac{2ab}{a^2-b^2}}$, shew that $\dfrac{ab}{a^2+b^2}\left(x^{\frac{a}{b}}+x^{\frac{b}{a}}\right)=\left(\dfrac{a}{b}\right)^{\frac{a^2+b^2}{a^2-b^2}}$.

[EMM. COLL. CAMB.]

62. Shew that the coefficient of x^{4n} in the expansion of

$$(1-x+x^2-x^3)^{-1} \text{ is unity.}$$

63. Solve the equation

$$\frac{x-a}{b}+\frac{x-b}{a}=\frac{b}{x-a}+\frac{a}{x-b}.$$

[LONDON UNIVERSITY.]

64. Find (1) the arithmetical series, (2) the harmonical series of n terms of which a and b are the first and last terms; and shew that the product of the r^{th} term of the first series and the $(n-r+1)^{\text{th}}$ term of the second series is ab.

65. If the roots of the equation

$$\left(1-q+\frac{p^2}{2}\right)x^2+p(1+q)x+q(q-1)+\frac{p^2}{2}=0$$

are equal, shew that $p^2=4q$. [R. M. A. Woolwich.]

66. If $a^2+b^2=7ab$, shew that

$$\log\left\{\frac{1}{3}(a+b)\right\}=\frac{1}{2}(\log a+\log b).$$

[Queen's Coll. Ox.]

67. If n is a root of the equation

$$x^2(1-ac)-x(a^2+c^2)-(1+ac)=0,$$

and if n harmonic means are inserted between a and c, shew that the difference between the first and last mean is equal to $ac(a-c)$.

[Wadham Coll. Ox.]

68. If ${}^{n+2}C_8 : {}^{n-2}P_4=57 : 16$, find n.

69. A person invests a certain sum in a $6\frac{1}{2}$ per cent. Government loan: if the price had been £3 less he would have received $\frac{1}{8}$ per cent. more interest on his money; at what price was the loan issued?

70. Solve the equation:

$$\{(x^2+x+1)^3-(x^2+1)^3-x^3\}\{(x^2-x+1)^3-(x^2+1)^3+x^3\}$$
$$=3\{(x^4+x^2+1)^3-(x^4+1)^3-x^6\}.$$

[Merton Coll. Ox.]

71. If by eliminating x between the equations

$$x^2+ax+b=0 \quad\text{and}\quad xy+l(x+y)+m=0,$$

a quadratic in y is formed whose roots are the same as those of the original quadratic in x, then either $a=2l$, and $b=m$, or $b+m=al$.

[R. M. A. Woolwich.]

72. Given $\log 2=\cdot30103$, and $\log 3=\cdot47712$, solve the equations:

(1) $6^x=\dfrac{10}{3}-6^{-x}$. (2) $\sqrt{5^x}+\sqrt{5^{-x}}=\dfrac{29}{10}$.

73. Find two numbers such that their sum is 9, and the sum of their fourth powers 2417. [London University.]

74. A set out to walk at the rate of 4 miles an hour; after he had been walking $2\frac{3}{4}$ hours, B set out to overtake him and went $4\frac{1}{2}$ miles the first hour, $4\frac{3}{4}$ miles the second, 5 the third, and so gaining a quarter of a mile every hour. In how many hours would he overtake A?

75. Prove that the integer next above $(\sqrt{3}+1)^{2m}$ contains 2^{m+1} as a factor.

76. The series of natural numbers is divided into groups 1; 2, 3, 4; 5, 6, 7, 8, 9; and so on: prove that the sum of the numbers in the n^{th} group is $(n-1)^3+n^3$.

77. Shew that the sum of n terms of the series

$$\frac{1}{2}+\frac{1}{\underline{|2}}\left(\frac{1}{2}\right)^2+\frac{1.3}{\underline{|3}}\left(\frac{1}{2}\right)^3+\frac{1.3.5}{\underline{|4}}\left(\frac{1}{2}\right)^4+\dots\dots$$

is equal to $1-\dfrac{1.3.5.7\dots\dots(2n-1)}{2^n\ \underline{|n}}$.

[R. M. A. Woolwich.]

78. Shew that the coefficient of x^n in the expansion of $\dfrac{1+2x}{1-x+x^2}$ is

$$(-1)^{\frac{n}{3}},\ 3(-1)^{\frac{n-1}{3}},\ 2(-1)^{\frac{n-2}{3}},$$

according as n is of the form $3m$, $3m+1$, $3m+2$.

79. Solve the equations:

$$(1)\ \frac{x}{a}=\frac{y}{b}=\frac{z}{c}=\frac{xyz}{x+y+z}.$$

$$(2)\ \frac{x}{y}+\frac{y}{z}+\frac{z}{x}=\frac{y}{x}+\frac{z}{y}+\frac{x}{z}=x+y+z=3.$$

[Univ. Coll. Ox.]

80. The value of xyz is $7\frac{1}{2}$ or $3\frac{3}{5}$ according as the series a, x, y, z, b is arithmetic or harmonic: find the values of a and b assuming them to be positive integers. [Merton Coll. Ox.]

81. If $ay-bx=c\sqrt{(x-a)^2+(y-b)^2}$, shew that x and y are connected by a linear relation if $c^2 \leqslant a^2+b^2$.

82. If $(x+1)^2$ is greater than $5x-1$ and less than $7x-3$, find the integral value of x.

83. If P is the number of integers whose logarithms have the characteristic p, and Q the number of integers the logarithms of whose reciprocals have the characteristic $-q$, shew that

$$\log_{10}P-\log_{10}Q=p-q+1.$$

84. In how many ways may 20 shillings be given to 5 persons so that no person may receive less than 3 shillings?

85. A man wishing his two daughters to receive equal portions when they came of age bequeathed to the elder the accumulated interest of a certain sum of money invested at the time of his death in 4 per cent. stock at 88; and to the younger he bequeathed the accumulated interest of a sum less than the former by £3500 invested at the same time in the 3 per cents. at 63. Supposing their ages at the time of their father's death to have been 17 and 14, what was the sum invested in each case, and what was each daughter's fortune?

86. A number of three digits in scale 7 when expressed in scale 9 has its digits reversed in order: find the number.

[ST JOHN'S COLL. CAMB.]

87. If the sum of m terms of an arithmetical progression is equal to the sum of either the next n terms or the next p terms, prove that $(m+n)\left(\frac{1}{m}-\frac{1}{p}\right)=(m+p)\left(\frac{1}{m}-\frac{1}{n}\right)$.

[ST JOHN'S COLL. CAMB.]

88. Prove that

$$\frac{1}{(y-z)^2}+\frac{1}{(z-x)^2}+\frac{1}{(x-y)^2}=\left(\frac{1}{y-z}+\frac{1}{z-x}+\frac{1}{x-y}\right)^2.$$

[R. M. A. WOOLWICH.]

89. If m is negative, or positive and greater than 1, shew that

$$1^m+3^m+5^m+\ldots\ldots+(2n-1)^m>n^{m+1}.$$

[EMM. COLL. CAMB.]

90. If each pair of the three equations

$$x^2-p_1x+q_1=0,\quad x^2-p_2x+q_2=0,\quad x^2-p_3x+q_3=0,$$

have a common root, prove that

$$p_1^2+p_2^2+p_3^2+4(q_1+q_2+q_3)=2(p_2p_3+p_3p_1+p_1p_2).$$

[ST JOHN'S COLL. CAMB.]

91. A and B travelled on the same road and at the same rate from Huntingdon to London. At the 50^{th} milestone from London, A overtook a drove of geese which were proceeding at the rate of 3 miles in 2 hours; and two hours afterwards met a waggon, which was moving at the rate of 9 miles in 4 hours. B overtook the same drove of geese at the 45^{th} milestone, and met the waggon exactly 40 minutes before he came to the 31^{st} milestone. Where was B when A reached London?

[ST JOHN'S COLL. CAMB.]

92. If $a+b+c+d=0$, prove that

$$abc+bcd+cda+dab=\sqrt{(bc-ad)(ca-bd)(ab-cd)}.$$

[R. M. A. WOOLWICH.]

93. An A. P., a G. P., and an H. P. have a and b for their first two terms: shew that their $(n+2)^{\text{th}}$ terms will be in G. P. if

$$\frac{b^{2n+2}-a^{2n+2}}{ba(b^{2n}-a^{2n})}=\frac{n+1}{n}.$$

[MATH. TRIPOS.]

94. Shew that the coefficient of x^n in the expansion of $\frac{x}{(x-a)(x-b)}$ in ascending power of x is $\frac{a^n-b^n}{a-b}\cdot\frac{1}{a^nb^n}$; and that the coefficient of x^{3n} in the expansion of $\frac{(1+x^2)^n}{(1-x)^3}$ is $2^{n-1}(n^2+4n+2)$. [EMM. COLL. CAMB.]

95. Solve the equations:

$$\sqrt{x-y}+\frac{1}{2}\sqrt{x+y}=\frac{x-1}{\sqrt{x-y}}, \quad x^2+y^2 : xy=34 : 15.$$

[ST JOHN'S COLL. CAMB.]

96. Find the value of $1+\frac{1}{3+}\frac{1}{2+}\frac{1}{3+}\frac{1}{2+}\dots$ in the form of a quadratic surd. [R. M. A. WOOLWICH.]

97. Prove that the cube of an integer may be expressed as the difference of two squares; that the cube of every odd integer may be so expressed in two ways; and that the difference of the cubes of any two consecutive integers may be expressed as the difference of two squares. [JESUS COLL. CAMB.]

98. Find the value of the infinite series

$$\frac{1}{\underline{|3}}+\frac{2}{\underline{|5}}+\frac{3}{\underline{|7}}+\frac{4}{\underline{|9}}+\dots$$ [EMM. COLL. CAMB.]

99. If $$x=\frac{a}{b+}\frac{c}{d+}\frac{a}{b+}\frac{c}{d+}\dots\dots,$$

and $$y=\frac{c}{d+}\frac{a}{b+}\frac{c}{d+}\frac{a}{b+}\dots\dots,$$

then $$bx-dy=a-c.$$ [CHRIST'S COLL. CAMB.]

100. Find the generating function, the sum to n terms, and the n^{th} term of the recurring series $1+5x+7x^2+17x^3+31x^4+\dots\dots$

101. If a, b, c are in H. P., then

(1) $\frac{a+b}{2a-b}+\frac{c+b}{2c-b}>4.$

(2) $b^2(a-c)^2=2\{c^2(b-a)^2+a^2(c-b)^2\}.$ [PEMB. COLL. CAMB.]

102. If a, b, c are all real quantities, and $x^3-3b^2x+2c^3$ is divisible by $x-a$ and also by $x-b$; prove that either $a=b=c$, or $a=-2b=-2c$. [JESUS COLL. OX.]

103. Shew that the sum of the squares of three consecutive odd numbers increased by 1 is divisible by 12, but not by 24.

104. Shew that $\frac{ac-b^2}{a}$ is the greatest or least value of $ax^2+2bx+c$, according as a is negative or positive.

If $x^4+y^4+z^4+y^2z^2+z^2x^2+x^2y^2=2xyz(x+y+z)$, and x, y, z are all real, shew that $x=y=z$. [ST JOHN'S COLL. CAMB.]

105. Shew that the expansion of $\sqrt{\dfrac{1-\sqrt{1-x^2}}{2}}$

is
$$\frac{x}{2}+\frac{1.3}{2.4}\cdot\frac{x^3}{6}+\frac{1.3.5.7}{2.4.6.8}\cdot\frac{x^5}{10}+\ldots\ldots$$

106. If a, β are roots of the equations

$$x^2+px+q=0,\quad x^{2n}+p^nx^n+q^n=0,$$

where n is an even integer, shew that $\frac{a}{\beta}$, $\frac{\beta}{a}$ are roots of

$$x^n+1+(x+1)^n=0.$$ [PEMB. COLL. CAMB.]

107. Find the difference between the squares of the infinite continued fractions

$$a+\frac{b}{2a+}\,\frac{b}{2a+}\,\frac{b}{2a+}\ldots\ldots,\quad\text{and } c+\frac{d}{2c+}\,\frac{d}{2c+}\,\frac{d}{2c+}\ldots\ldots$$

[CHRIST'S COLL. CAMB.]

108. A sum of money is distributed amongst a certain number of persons. The second receives 1*s.* more than the first, the third 2*s.* more than the second, the fourth 3*s.* more than the third, and so on. If the first person gets 1*s.* and the last person £3. 7*s.*, what is the number of persons and the sum distributed?

109. Solve the equations:

(1) $$\frac{x}{a}+\frac{y+z}{b+c}=\frac{y}{b}+\frac{z+x}{c+a}=\frac{z}{c}+\frac{x+y}{a+b}=2.$$

(2) $$\frac{x^2+y^2}{xy}+x^2+y^2=13\tfrac{1}{3},\qquad \frac{xy}{x^2+y^2}+xy=3\tfrac{3}{10}.$$

110. If $a>b>0$, and n is a positive integer, prove that

$$a^n-b^n>n(a-b)(ab)^{\frac{n-1}{2}}.$$

[ST CATH. COLL. CAMB.]

111. Express $\frac{763}{396}$ as a continued fraction; hence find the least positive integral values of x and y which satisfy the equation

$$396x-763y=12.$$

112. To complete a certain work, a workman A alone would take m times as many days as B and C working together; B alone would take n times as many days as A and C together; C alone would take p times as many days as A and B together: shew that the numbers of days in which each would do it alone are as $m+1:n+1:p+1$.

Prove also $\frac{m}{m+1}+\frac{n}{n+1}+\frac{p}{p+1}=2.$ [R. M. A. WOOLWICH.]

113. The expenses of a hydropathic establishment are partly constant and partly vary with the number of boarders. Each boarder pays £65 a year, and the annual profits are £9 a head when there are 50 boarders, and £10. 13*s*. 4*d*. when there are 60: what is the profit on each boarder when there are 80?

114. If $x^2y=2x-y$, and x^2 is not greater than 1, shew that

$$4\left(x^2+\frac{x^6}{3}+\frac{x^{10}}{5}+\dots\right)=y^2+\frac{y^4}{2}+\frac{y^6}{3}+\dots$$

[PETERHOUSE, CAMB.]

115. If $\frac{x}{a^2-y^2}=\frac{y}{a^2-x^2}=\frac{1}{b}$, and $xy=c^2$, shew that when a and c are unequal,

$$(a^2-c^2)^2-b^2c^2=0, \text{ or } a^2+c^2-b^2=0.$$

116. If $$(1+x+x^2)^{3r}=1+k_1x+k_2x^2+\dots,$$

and $$(x-1)^{3r}=x^{3r}-c_1x^{3r-1}+c_2x^{3r-2}-\dots;$$

prove that (1) $1-k_1+k_2-\dots\dots=1$,

(2) $1-k_1c_1+k_2c_2-\dots\dots=\pm\frac{\lfloor 3r}{\lfloor r\,\lfloor 2r}$.

[R. M. A. WOOLWICH.]

117. Solve the equations:

(1) $(x-y)^2+2ab=ax+by,\quad xy+ab=bx+ay.$

(2) $x^2-y^2+z^2=6,\quad 2yz-zx+2xy=13,\quad x-y+z=2.$

118. If there are n positive quantities $a_1, a_2, \dots a_n$, and if the square roots of all their products taken two together be found, prove that

$$\sqrt{a_1a_2}+\sqrt{a_1a_3}+\dots\dots<\frac{n-1}{2}(a_1+a_2+\dots\dots+a_n);$$

hence prove that the arithmetic mean of the square roots of the products two together is less than the arithmetic mean of the given quantities. [R. M. A. WOOLWICH.]

119. If $b^2x^4+a^2y^4=a^2b^2$, and $a^2+b^2=x^2+y^2=1$, prove that

$$b^4x^6+a^4y^6=(b^2x^4+a^2y^4)^2.$$ [INDIA CIVIL SERVICE.]

120. Find the sum of the first n terms of the series whose r^{th} terms are (1) $\frac{2r+1}{r^2(r+1)^2}$, (2) $(a+r^2b)\,x^{n-r}$.

[ST JOHN'S COLL. CAMB.]

121. Find the greatest value of $\frac{x+2}{2x^2+3x+6}$.

122. Solve the equations:

(1) $1+x^4=7(1+x)^4$.

(2) $3xy+2z=xz+6y=2yz+3x=0$.

123. If a_1, a_2, a_3, a_4 are any four consecutive coefficients of an expanded binomial, prove that

$$\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}.$$ [QUEENS' COLL. CAMB.]

124. Separate $\dfrac{x^3+7x^2-x-8}{(x^2+x+1)(x^2-3x-1)}$ into partial fractions; and find the general term when $(3x-8)/(x^2-4x+4)$ is expanded in ascending powers of x.

125. In the recurring series

$$\frac{5}{4}-\frac{1}{2}x+2x^2+lx^3+5x^4+7x^5+\ldots\ldots$$

the scale of relation is a quadratic expression; determine the unknown coefficient of the fourth term and the scale of relation, and give the general term of the series. [R. M. A. WOOLWICH.]

126. If x, y, z are unequal, and if

$$2a-3y=\frac{(z-x)^2}{y}, \text{ and } 2a-3z=\frac{(x-y)^2}{z},$$

then will $$2a-3x=\frac{(y-z)^2}{x}, \text{ and } x+y+z=a.$$ [MATH. TRIPOS.]

127. Solve the equations:

(1) $xy+6=2x-x^2$, $xy-9=2y-y^2$.

(2) $(ax)^{\log a}=(by)^{\log b}$, $b^{\log x}=a^{\log y}$.

128. Find the limiting values of

(1) $x\sqrt{x^2+a^2}-\sqrt{x^4+a^4}$, when $x=\infty$.

(2) $\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$, when $x=a$. [LONDON UNIVERSITY.]

129. There are two numbers whose product is 192, and the quotient of the arithmetical by the harmonical mean of their greatest common measure and least common multiple is $3\frac{25}{48}$: find the numbers. [R. M. A. WOOLWICH.]

130. Solve the following equations:

(1) $\sqrt[3]{13x+37}-\sqrt[3]{13x-37}=\sqrt[3]{2}$.

(2) $$\left.\begin{aligned} b\sqrt{1-z^2}+c\sqrt{1-y^2}&=a,\\ c\sqrt{1-x^2}+a\sqrt{1-z^2}&=b,\\ a\sqrt{1-y^2}+b\sqrt{1-x^2}&=c. \end{aligned}\right\}$$

131. Prove that the sum to infinity of the series

$$\frac{1}{2^3\,\underline{|3}}-\frac{1.3}{2^4\,\underline{|4}}+\frac{1.3.5}{2^5\,\underline{|5}}-\ldots \text{ is } \frac{23}{24}-\frac{2}{3}\sqrt{2}.$$ [MATH. TRIPOS.]

132. A number consisting of three digits is doubled by reversing the digits; prove that the same will hold for the number formed by the first and last digits, and also that such a number can be found in only one scale of notation out of every three. [MATH. TRIPOS.]

133. Find the coefficients of x^{12} and x^r in the product of

$$\frac{1+x^3}{(1-x^2)(1-x)} \text{ and } 1-x+x^2.$$ [R. M. A. WOOLWICH.]

134. A purchaser is to take a plot of land fronting a street; the plot is to be rectangular, and three times its frontage added to twice its depth is to be 96 yards. What is the greatest number of square yards he may take? [LONDON UNIVERSITY.]

135. Prove that

$$\begin{aligned}&(a+b+c+d)^4+(a+b-c-d)^4+(a-b+c-d)^4+(a-b-c+d)^4\\ &-(a+b+c-d)^4-(a+b-c+d)^4-(a-b+c+d)^4-(-a+b+c+d)^4\\ &=192abcd.\end{aligned}$$

[TRIN. COLL. CAMB.]

136. Find the values of a, b, c which will make each of the expressions $x^4+ax^3+bx^2+cx+1$ and $x^4+2ax^3+2bx^2+2cx+1$ a perfect square. [LONDON UNIVERSITY.]

137. Solve the equations:

(1) $\dfrac{\sqrt[3]{x+y}-\sqrt[3]{x-y}}{\sqrt[3]{x+y}+\sqrt[3]{x-y}}=3,\quad x^2+y^2=65.$

(2) $\sqrt{2x^2+1}+\sqrt{2x^2-1}=\dfrac{2}{\sqrt{3-2x^2}}$.

138. A farmer sold 10 sheep at a certain price and 5 others at 10s. less per head; the sum he received for each lot was expressed in pounds by the same two digits: find the price per sheep.

139. Sum to n terms:

(1) $(2n-1)+2(2n-3)+3(2n-5)+\dots$.

(2) The squares of the terms of the series 1, 3, 6, 10, 15....

(3) The odd terms of the series in (2). [TRIN. COLL. CAMB.]

140. If α, β, γ are the roots of the equation $x^3+qx+r=0$ prove that $3(\alpha^2+\beta^2+\gamma^2)(\alpha^5+\beta^5+\gamma^5)=5(\alpha^3+\beta^3+\gamma^3)(\alpha^4+\beta^4+\gamma^4)$.
[ST JOHN'S COLL. CAMB.]

141. Solve the equations:

(1) $\left.\begin{aligned} x(3y-5)&=4 \\ y(2x+7)&=27 \end{aligned}\right\}$. (2) $\left.\begin{aligned} x^3+y^3+z^3&=495 \\ x+y+z&=15 \\ xyz&=105 \end{aligned}\right\}$.

[TRIN. COLL. CAMB.]

142. If a, b, c are the roots of the equation $x^3+qx^2+r=0$, form the equation whose roots are $a+b-c$, $b+c-a$, $c+a-b$.

143. Sum the series:

(1) $n+(n-1)x+(n-2)x^2+\dots+2x^{n-2}+x^{n-1}$;

(2) $3-x-2x^2-16x^3-28x^4-676x^5+\dots$ to infinity;

(3) $6+9+14+23+40+\dots$ to n terms.

[OXFORD MODS.]

144. Eliminate x, y, z from the equations

$$x^{-1}+y^{-1}+z^{-1}=a^{-1}, \quad x+y+z=b.$$

$$x^2+y^2+z^2=c^2, \quad x^3+y^3+z^3=d^3,$$

and shew that if x, y, z are all finite and numerically unequal, b cannot be equal to d. [R. M. A. WOOLWICH.]

145. The roots of the equation $3x^2(x^2+8)+16(x^3-1)=0$ are not all unequal: find them. [R. M. A. WOOLWICH.]

146. A traveller set out from a certain place, and went 1 mile the first day, 3 the second, 5 the next, and so on, going every day 2 miles more than he had gone the preceding day. After he had been gone three days, a second sets out, and travels 12 miles the first day, 13 the second, and so on. In how many days will the second overtake the first? Explain the double answer.

147. Find the value of

$$\frac{1}{3+}\,\frac{1}{2+}\,\frac{1}{1+}\,\frac{1}{3+}\,\frac{1}{2+}\,\frac{1}{1+}\dots$$

148. Solve the equation

$$x^3+3ax^2+3(a^2-bc)x+a^3+b^3+c^3-3abc=0.$$

[INDIA CIVIL SERVICE.]

149. If n is a prime number which will divide neither a, b, nor $a+b$, prove that $a^{n-2}b-a^{n-3}b^2+a^{n-4}b^3-\ldots+ab^{n-2}$ exceeds by 1 a multiple of n. [ST JOHN'S COLL. CAMB.]

150. Find the n^{th} term and the sum to n terms of the series whose sum to infinity is $(1-abx^2)(1-ax)^{-2}(1-bx)^{-2}$.

[OXFORD MODS.]

151. If a, b, c are the roots of the equation $x^3+px+q=0$, find the equation whose roots are $\dfrac{b^2+c^2}{a}$, $\dfrac{c^2+a^2}{b}$, $\dfrac{a^2+b^2}{c}$.

[TRIN. COLL. CAMB.]

152. Prove that

$$(y+z-2x)^4+(z+x-2y)^4+(x+y-2z)^4=18(x^2+y^2+z^2-yz-zx-xy)^2.$$

[CLARE COLL. CAMB.]

153. Solve the equations:

(1) $x^3-30x+133=0$, by Cardan's method.

(2) $x^5-4x^4-10x^3+40x^2+9x-36=0$, having roots of the form $\pm a$, $\pm b$, c.

154. It is found that the quantity of work done by a man in an hour varies directly as his pay per hour and inversely as the square root of the number of hours he works per day. He can finish a piece of work in six days when working 9 hours a day at 1*s.* per hour. How many days will he take to finish the same piece of work when working 16 hours a day at 1*s.* 6*d.* per hour?

155. If s_n denote the sum to n terms of the series

$$1.2+2.3+3.4+\ldots,$$

and σ_{n-1} that to $n-1$ terms of the series

$$\frac{1}{1.2.3.4}+\frac{1}{2.3.4.5}+\frac{1}{3.4.5.6}+\ldots,$$

shew that $$18s_n\sigma_{n-1}-s_n+2=0.$$

[MAGD. COLL. OX.]

156. Solve the equations:

(1) $(12x-1)(6x-1)(4x-1)(3x-1)=5.$

(2) $\dfrac{1}{5}\dfrac{(x+1)(x-3)}{(x+2)(x-4)}+\dfrac{1}{9}\dfrac{(x+3)(x-5)}{(x+4)(x-6)}-\dfrac{2}{13}\dfrac{(x+5)(x-7)}{(x+6)(x-8)}=\dfrac{92}{585}.$

[ST JOHN'S COLL. CAMB.]

157. A cottage at the beginning of a year was worth £250, but it was found that by dilapidations at the end of each year it lost ten per cent. of the value it had at the beginning of each year: after what number of years would the value of the cottage be reduced below £25? Given $\log_{10} 3 = \cdot 4771213$. [R. M. A. WOOLWICH.]

158. Shew that the infinite series

$$1+\frac{1}{4}+\frac{1.4}{4.8}+\frac{1.4.7}{4.8.12}+\frac{1.4.7.10}{4.8.12.16}+\dots,$$

$$1+\frac{2}{6}+\frac{2.5}{6.12}+\frac{2.5.8}{6.12.18}+\frac{2.5.8.11}{6.12.18.24}+\dots,$$

are equal. [PETERHOUSE, CAMB.]

159. Prove the identity

$$\left\{1-\frac{x}{\alpha}+\frac{x(x-\alpha)}{\alpha\beta}-\frac{x(x-\alpha)(x-\beta)}{\alpha\beta\gamma}+\dots\right\}$$
$$\left\{1+\frac{x}{\alpha}+\frac{x(x+\alpha)}{\alpha\beta}+\frac{x(x+\alpha)(x+\beta)}{\alpha\beta\gamma}+\dots\right\}$$
$$=1-\frac{x^2}{\alpha^2}+\frac{x^2(x^2-\alpha^2)}{\alpha^2\beta^2}-\frac{x^2(x^2-\alpha^2)(x^2-\beta^2)}{\alpha^2\beta^2\gamma^2}+\dots.$$

[TRIN. COLL. CAMB.]

160. If n is a positive integer greater than 1, shew that

$$n^5-5n^3+60n^2-56n$$

is a multiple of 120. [WADHAM COLL. OX.]

161. A number of persons were engaged to do a piece of work which would have occupied them 24 hours if they had commenced at the same time; but instead of doing so, they commenced at equal intervals and then continued to work till the whole was finished, the payment being proportional to the work done by each: the first comer received eleven times as much as the last; find the time occupied.

162. Solve the equations:

(1) $$\frac{x}{y^2-3}=\frac{y}{x^2-3}=\frac{-7}{x^3+y^3}.$$

(2) $$y^2+z^2-x(y+z)=a^2,$$
$$z^2+x^2-y(z+x)=b^2,$$
$$x^2+y^2-z(x+y)=c^2.$$

[PEMB. COLL. CAMB.]

163. Solve the equation

$$a^3(b-c)(x-b)(x-c)+b^3(c-a)(x-c)(x-a)+c^3(a-b)(x-a)(x-b)=0;$$

also shew that if the two roots are equal

$$\frac{1}{\sqrt{a}}\pm\frac{1}{\sqrt{b}}\pm\frac{1}{\sqrt{c}}=0.$$ [ST JOHN'S COLL. CAMB.]

164. Sum the series:

(1) $1.2.4+2.3.5+3.4.6+\ldots$ to n terms.

(2) $\frac{1^2}{\underline{|3}}+\frac{2^2}{\underline{|4}}+\frac{3^2}{\underline{|5}}+\ldots$ to inf.

165. Shew that, if a, b, c, d be four positive unequal quantities and $s=a+b+c+d$, then

$$(s-a)(s-b)(s-c)(s-d)>81abcd.$$

[PETERHOUSE, CAMB.]

166. Solve the equations:

(1) $\sqrt{x+a}-\sqrt{y-a}=\frac{5}{2}\sqrt{a},\ \sqrt{x-a}-\sqrt{y+a}=\frac{3}{2}\sqrt{a}.$

(2) $x+y+z=x^2+y^2+z^2=\frac{1}{2}(x^3+y^3+z^3)=3.$

[MATH. TRIPOS.]

167. Eliminate l, m, n from the equations:

$$lx+my+nz=mx+ny+lz=nx+ly+mz=k^2(l^2+m^2+n^2)=1.$$

168. Simplify

$$\frac{a(b+c-a)^2+\ldots+\ldots+(b+c-a)(c+a-b)(a+b-c)}{a^2(b+c-a)+\ldots+\ldots-(b+c-a)(c+a-b)(a+b-c)}.$$

[MATH. TRIPOS.]

169. Shew that the expression

$$(x^2-yz)^3+(y^2-zx)^3+(z^2-xy)^3-3(x^2-yz)(y^2-zx)(z^2-xy)$$

is a perfect square, and find its square root. [LONDON UNIVERSITY.]

170. There are three towns A, B, and C; a person by walking from A to B, driving from B to C, and riding from C to A makes the journey in $15\frac{1}{2}$ hours; by driving from A to B, riding from B to C, and walking from C to A he could make the journey in 12 hours. On foot he could make the journey in 22 hours, on horseback in $8\frac{1}{4}$ hours, and driving in 11 hours. To walk a mile, ride a mile, and drive a mile he takes altogether half an hour: find the rates at which he travels, and the distances between the towns.

171. Shew that $n^7-7n^5+14n^3-8n$ is divisible by 840, if n is an integer not less than 3.

172. Solve the equations:

(1) $\sqrt{x^2+12y}+\sqrt{y^2+12x}=33, \quad x+y=23.$

(2) $\dfrac{u(y-x)}{z-u}=a, \quad \dfrac{z(y-x)}{z-u}=b, \quad \dfrac{y(u-z)}{x-y}=c, \quad \dfrac{x(u-z)}{x-y}=d.$

[MATH. TRIPOS.]

173. If s be the sum of n positive unequal quantities $a, b, c, \ldots$, then

$$\frac{s}{s-a}+\frac{s}{s-b}+\frac{s}{s-c}+\ldots>\frac{n^2}{n-1}.$$ [MATH. TRIPOS.]

174. A merchant bought a quantity of cotton; this he exchanged for oil which he sold. He observed that the number of cwt. of cotton, the number of gallons of oil obtained for each cwt., and the number of shillings for which he sold each gallon formed a descending geometrical progression. He calculated that if he had obtained one cwt. more of cotton, one gallon more of oil for each cwt., and 1*s.* more for each gallon, he would have obtained £508. 9*s.* more; whereas if he had obtained one cwt. less of cotton, one gallon less of oil for each cwt., and 1*s.* less for each gallon, he would have obtained £483. 13*s.* less: how much did he actually receive?

175. Prove that

$$\Sigma(b+c-a-x)^4(b-c)(a-x)=16(b-c)(c-a)(a-b)(x-a)(x-b)(x-c).$$

[JESUS COLL. CAMB.]

176. If α, β, γ are the roots of the equation $x^3-px^2+r=0$, find the equation whose roots are $\dfrac{\beta+\gamma}{\alpha}, \dfrac{\gamma+\alpha}{\beta}, \dfrac{\alpha+\beta}{\gamma}$. [R. M. A. WOOLWICH.]

177. If any number of factors of the form a^2+b^2 are multiplied together, shew that the product can be expressed as the sum of two squares.

Given that $(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2)=p^2+q^2$, find p and q in terms of a, b, c, d, e, f, g, h. [LONDON UNIVERSITY.]

178. Solve the equations

$$x^2+y^2=61, \quad x^3-y^3=91.$$ [R. M. A. WOOLWICH.]

179. A man goes in for an Examination in which there are four papers with a maximum of m marks for each paper; shew that the number of ways of getting $2m$ marks on the whole is

$$\frac{1}{3}(m+1)(2m^2+4m+3).$$ [MATH. TRIPOS.]

180. If α, β are the roots of $x^2+px+1=0$, and γ, δ are the roots of $x^2+qx+1=0$; shew that $(\alpha-\gamma)(\beta-\gamma)(\alpha+\delta)(\beta+\delta)=q^2-p^2$.

[R. M. A. WOOLWICH.]

181. Shew that if a_m be the coefficient of x^m in the expansion of $(1+x)^n$, then whatever n be,

$$a_0-a_1+a_2-\ldots+(-1)^{m-1}a_{m-1}=\frac{(n-1)(n-2)\ldots(n-m+1)}{\lfloor m-1}(-1)^{m-1}.$$

[NEW COLL. OX.]

182. A certain number is the product of three prime factors, the sum of whose squares is 2331. There are 7560 numbers (including unity) which are less than the number and prime to it. The sum of its divisors (including unity and the number itself) is 10560. Find the number. [CORPUS COLL. CAMB.]

183. Form an equation whose roots shall be the products of every two of the roots of the equation $x^3-ax^2+bx+c=0$.

Solve completely the equation

$$2x^5+x^4+x+2=12x^3+12x^2.$$

[R. M. A. WOOLWICH.]

184. Prove that if n is a positive integer,

$$n^n-n(n-2)^n+\frac{n(n-1)}{\lfloor 2}(n-4)^n-\ldots\ldots=2^n\lfloor n.$$

185. If $(6\sqrt{6}+14)^{2n+1}=N$, and if F be the fractional part of N, prove that $NF=20^{2n+1}$. [EMM. COLL. CAMB.]

186. Solve the equations:

(1) $x+y+z=2,\ x^2+y^2+z^2=0,\ x^3+y^3+z^3=-1$.

(2) $x^2-(y-z)^2=a^2,\ y^2-(z-x)^2=b^2,\ z^2-(x-y)^2=c^2$.

[CHRIST'S COLL. CAMB.]

187. At a general election the whole number of Liberals returned was 15 more than the number of English Conservatives, the whole number of Conservatives was 5 more than twice the number of English Liberals. The number of Scotch Conservatives was the same as the number of Welsh Liberals, and the Scotch Liberal majority was equal to twice the number of Welsh Conservatives, and was to the Irish Liberal majority as 2 : 3. The English Conservative majority was 10 more than the whole number of Irish members. The whole number of members was 652, of whom 60 were returned by Scotch constituencies. Find the numbers of each party returned by England, Scotland, Ireland, and Wales, respectively. [ST JOHN'S COLL. CAMB.]

188. Shew that $a^5(c-b)+b^5(a-c)+c^5(b-a)$

$$=(b-c)(c-a)(a-b)(\Sigma a^3+\Sigma a^2b+abc).$$

189. Prove that $\begin{vmatrix} a^3 & 3a^2 & 3a & 1 \\ a^2 & a^2+2a & 2a+1 & 1 \\ a & 2a+1 & a+2 & 1 \\ 1 & 3 & 3 & 1 \end{vmatrix} = (a-1)^6.$ [BALL. COLL. OX.]

190. If $\dfrac{1}{a}+\dfrac{1}{c}+\dfrac{1}{a-b}+\dfrac{1}{c-b}=0$, prove that a, b, c are in harmonical progression, unless $b=a+c$. [TRIN. COLL. CAMB.]

191. Solve the equations:

(1) $x^3-13x^2+15x+189=0$, having given that one root exceeds another root by 2.

(2) $x^4-4x^3+8x+35=0$, having given that one root is $2+\sqrt{-3}$. [R. M. A. WOOLWICH.]

192. Two numbers a and b are given; two others a_1, b_1 are formed by the relations $3a_1=2a+b$, $3b_1=a+2b$; two more a_2, b_2 are formed from a_1, b_1 in the same manner, and so on; find a_n, b_n in terms of a and b, and prove that when n is infinite, $a_n=b_n$. [R. M. A. WOOLWICH.]

193. If $x+y+z+w=0$, shew that

$$wx(w+x)^2+yz(w-x)^2+wy(w+y)^2$$
$$+zx(w-y)^2+wz(w+z)^2+xy(w-z)^2+4xyzw=0.$$

[MATH. TRIPOS.]

194. If $a+\dfrac{bc-a^2}{a^2+b^2+c^2}$ be not altered in value by interchanging a and b (a, b and c being unequal), it will not be altered by interchanging a and c, and vice versa; and it will vanish if $a+b+c=1$.

[MATH. TRIPOS.]

195. On a quadruple line of rails between two termini A and B, two down trains start at 6.0 and 6.45, and two up trains at 7.15 and 8.30. If the four trains (regarded as points) all pass one another simultaneously, find the following equations between x_1, x_2, x_3, x_4, their rates in miles per hour,

$$\frac{3x_2}{x_2-x_1}=\frac{4m+5x_3}{x_1+x_3}=\frac{4m+10x_4}{x_1+x_4},$$

where m is the number of miles in AB. [TRIN. COLL. CAMB.]

196. Prove that, rejecting terms of the third and higher orders,

$$\frac{(1-x)^{-\frac{1}{2}}+(1-y)^{-\frac{1}{2}}}{1+\sqrt{(1-x)(1-y)}}=1+\frac{1}{2}(x+y)+\frac{1}{8}(3x^2+xy+3y^2).$$

[TRIN. COLL. CAMB.]

197. Shew that the sum of the products of the series

$$a,\ a-b,\ a-2b,\ \ldots\ldots,\ a-(n-1)b,$$

taken two and two together vanishes when n is of the form $3m^2-1$, and $2a=(3m-2)(m+1)b$.

198. If n is even, and $\alpha+\beta$, $\alpha-\beta$ are the middle pair of terms, shew that the sum of the cubes of an arithmetical progression is

$$n\alpha\{\alpha^2+(n^2-1)\beta^2\}.$$

199. If a, b, c are real positive quantities, shew that

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}<\frac{a^8+b^8+c^8}{a^3b^3c^3}.$$

[TRIN. COLL. CAMB.]

200. A, B, and C start at the same time for a town a miles distant; A walks at a uniform rate of u miles an hour, and B and C drive at a uniform rate of v miles an hour. After a certain time B dismounts and walks forward at the same pace as A, while C drives back to meet A; A gets into the carriage with C and they drive after B entering the town at the same time that he does: shew that the whole time occupied was $\frac{a}{v}\cdot\frac{3v+u}{3u+v}$ hours. [PETERHOUSE, CAMB.]

201. The streets of a city are arranged like the lines of a chess-board. There are m streets running north and south, and n east and west. Find the number of ways in which a man can travel from the N.W. to the S.E. corner, going the shortest possible distance.

[OXFORD MODS.]

202. Solve the equation $\sqrt[4]{x+27}+\sqrt[4]{55-x}=4.$

[BALL. COLL. OX.]

203. Shew that in the series

$$ab+(a+x)(b+x)+(a+2x)(b+2x)+\ldots\ldots \text{ to } 2n \text{ terms,}$$

the excess of the sum of the last n terms over the sum of the first n terms is to the excess of the last term over the first as n^2 to $2n-1$.

204. Find the n^{th} convergent to

(1) $\frac{1}{2-}\ \frac{1}{2-}\ \frac{1}{2-}\ldots\ldots$

(2) $\frac{4}{3+}\ \frac{4}{3+}\ \frac{4}{3+}\ldots\ldots$

205. Prove that

$$\begin{aligned}(a-x)^4(y-z)^4+(a-y)^4(z-x)^4+(a-z)^4(x-y)^4\\ =2\{(a-y)^2(a-z)^2(x-y)^2(x-z)^2+(a-z)^2(a-x)^2(y-z)^2(y-x)^2\\ +(a-x)^2(a-y)^2(z-x)^2(z-y)^2\}.\end{aligned}$$

[PETERHOUSE, CAMB.]

206. If α, β, γ are the roots of $x^3+qx+r=0$, find the value of

$$\frac{m\alpha+n}{m\alpha-n}+\frac{m\beta+n}{m\beta-n}+\frac{m\gamma+n}{m\gamma-n}$$

in terms of m, n, q, r. [QUEENS' COLL. CAMB.]

207. In England one person out of 46 is said to die every year, and one out of 33 to be born. If there were no emigration, in how many years would the population double itself at this rate? Given

$$\log 2 = \cdot 3010300,\ \log 1531 = 3\cdot 1849752,\ \log 1518 = 3\cdot 1812718.$$

208. If $(1+x+x^2)^n = a_0 + a_1 x + a_2 x^2 + \ldots\ldots$, prove that

$$a_r - n a_{r-1} + \frac{n(n-1)}{1.2} a_{r-2} - \ldots\ldots + (-1)^r \frac{n!}{r!\,(n-r)!} a_0 = 0,$$

unless r is a multiple of 3. What is its value in this case?

[ST JOHN'S COLL. CAMB.]

209. In a mixed company consisting of Poles, Turks, Greeks, Germans and Italians, the Poles are one less than one-third of the number of Germans, and three less than half the number of Italians. The Turks and Germans outnumber the Greeks and Italians by 3; the Greeks and Germans form one less than half the company; while the Italians and Greeks form seven-sixteenths of the company: determine the number of each nation.

210. Find the sum to infinity of the series whose n^{th} term is

$$(n+1)n^{-1}(n+2)^{-1}(-x)^{n+1}.$$ [OXFORD MODS.]

211. If n is a positive integer, prove that

$$n - \frac{n(n^2-1)}{\lfloor 2} + \frac{n(n^2-1)(n^2-2^2)}{\lfloor 2 \lfloor 3} - \ldots\ldots$$
$$+(-1)^r \frac{n(n^2-1)(n^2-2^2)\ldots\ldots(n^2-r^2)}{\lfloor r \lfloor r+1} + \ldots\ldots = (-1)^{n+1}.$$

[PEMB. COLL. CAMB.]

212. Find the sum of the series:

(1) 6, 24, 60, 120, 210, 336,...... to n terms.

(2) $4 - 9x + 16x^2 - 25x^3 + 36x^4 - 49x^5 + \ldots\ldots$ to inf.

(3) $\frac{1.3}{2} + \frac{3.5}{2^2} + \frac{5.7}{2^3} + \frac{7.9}{2^4} + \ldots\ldots$ to inf.

213. Solve the equation

$$\begin{vmatrix} 4x & 6x+2 & 8x+1 \\ 6x+2 & 9x+3 & 12x \\ 8x+1 & 12x & 16x+2 \end{vmatrix} = 0.$$

[KING'S COLL. CAMB.]

214. Shew that

(1) $a^2(1+b^2)+b^2(1+c^2)+c^2(1+a^2)>6abc$;

(2) $n(a^{p+q}+b^{p+q}+c^{p+q}+\dots)>(a^p+b^p+c^p+\dots)(a^q+b^q+c^q+\dots)$,

the number of quantities a, b, c,... being n, and p and q being positive.

215. Solve the equations

$$\left.\begin{aligned} yz &= a(y+z)+a \\ zx &= a(z+x)+\beta \\ xy &= a(x+y)+\gamma \end{aligned}\right\}.$$

[TRIN. COLL. CAMB.]

216. If n be a prime number, prove that

$$1(2^{n-1}+1)+2\left(3^{n-1}+\frac{1}{2}\right)+3\left(4^{n-1}+\frac{1}{3}\right)+\dots+(n-1)\left(n^{n-1}+\frac{1}{n-1}\right)$$

is divisible by n. [QUEEN'S COLL. OX.]

217. In a shooting competition a man can score 5, 4, 3, 2, or 0 points for each shot: find the number of different ways in which he can score 30 in 7 shots. [PEMB. COLL. CAMB.]

218. Prove that the expression $x^5-bx^3+cx^2+dx-e$ will be the product of a complete square and a complete cube if

$$\frac{12b}{5}=\frac{9d}{b}=\frac{5e}{c}=\frac{d^2}{c^2}.$$

219. A bag contains 6 black balls and an unknown number, not greater than six, of white balls; three are drawn successively and not replaced and are all found to be white; prove that the chance that a black ball will be drawn next is $\frac{677}{909}$. [JESUS COLL. CAMB.]

220. Shew that the sum of the products of every pair of the squares of the first n whole numbers is $\frac{1}{360}n(n^2-1)(4n^2-1)(5n+6)$.

[CAIUS COLL. CAMB.]

221. If $\frac{a^2(b-c)}{x-a}+\frac{\beta^2(c-a)}{x-b}+\frac{\gamma^2(a-b)}{x-c}=0$ has equal roots, prove that $a(b-c)\pm\beta(c-a)\pm\gamma(a-b)=0$.

222. Prove that when n is a positive integer,

$$n=2^{n-1}-\frac{n-2}{1}2^{n-3}+\frac{(n-3)(n-4)}{\lfloor 2}2^{n-5}$$
$$-\frac{(n-4)(n-5)(n-6)}{\lfloor 3}2^{n-7}+\dots.$$

[CLARE COLL. CAMB.

223. Solve the equations:

(1) $x^2+2yz=y^2+2zx=z^2+2xy+3=76.$

(2) $$\left.\begin{aligned} x+y+z&=a+b+c \\ \frac{x}{a}+\frac{y}{b}+\frac{z}{c}&=3 \\ ax+by+cz&=bc+ca+ab \end{aligned}\right\}.$$

[CHRIST'S COLL. CAMB.]

224. Prove that if each of m points in one straight line be joined to each of n in another by straight lines terminated by the points, then, excluding the given points, the lines will intersect $\frac{1}{4}mn(m-1)(n-1)$ times. [MATH. TRIPOS.]

225. Having given $y=x+x^2+x^5$, expand x in the form

$$y+ay^2+by^3+cy^4+dy^5+\ldots\ldots;$$

and shew that $a^2d-3abc+2b^3=-1.$ [BALL. COLL. OX.]

226. A farmer spent three equal sums of money in buying calves, pigs, and sheep. Each calf cost £1 more than a pig and £2 more than a sheep; altogether he bought 47 animals. The number of pigs exceeded that of the calves by as many sheep as he could have bought for £9: find the number of animals of each kind.

227. Express log 2 in the form of the infinite continued fraction

$$\frac{1}{1+}\,\frac{1}{1+}\,\frac{2^2}{1+}\,\frac{3^2}{1+}\ldots\ldots\frac{n^2}{1+}\ldots. \qquad \text{[EULER.]}$$

228. In a certain examination six papers are set, and to each are assigned 100 marks as a maximum. Shew that the number of ways in which a candidate may obtain forty per cent. of the whole number of marks is

$$\frac{1}{5!}\left\{\frac{245!}{240!}-6\cdot\frac{144!}{139!}+15\cdot\frac{43!}{38!}\right\}. \qquad \text{[OXFORD MODS.]}$$

229. Test for convergency

$$\frac{x}{2}+\frac{1.3}{2.4}\cdot\frac{x^3}{6}+\frac{1.3.5.7}{2.4.6.8}\cdot\frac{x^5}{10}+\frac{1.3.5.7.9.11}{2.4.6.8.10.12}\cdot\frac{x^7}{14}+\ldots\ldots$$

230. Find the scale of relation, the n^{th} term, and the sum of n terms of the recurring series $1+6+40+288+\ldots\ldots$

Shew also that the sum of n terms of the series formed by taking for its r^{th} term the sum of r terms of this series is

$$\frac{2}{3^2}(2^{2n}-1)+\frac{4}{7^2}(2^{3n}-1)-\frac{5n}{21}. \qquad \text{[CAIUS COLL. CAMB.]}$$

231. It is known that at noon at a certain place the sun is hidden by clouds on an average two days out of every three: find the chance that at noon on at least four out of five specified future days the sun will be shining. [QUEEN'S COLL. OX.]

232. Solve the equations

$$\left.\begin{aligned} x^2+(y-z)^2&=a^2\\ y^2+(z-x)^2&=b^2\\ z^2+(x-y)^2&=c^2 \end{aligned}\right\}.$$

[EMM. COLL. CAMB.]

233. Eliminate x, y, z from the equations:

$$\frac{x^2-xy-xz}{a}=\frac{y^2-yz-yx}{b}=\frac{z^2-zx-zy}{c}, \text{ and } ax+by+cz=0.$$

[MATH. TRIPOS.]

234. If two roots of the equation $x^3+px^2+qx+r=0$ be equal and of opposite signs, shew that $pq=r$. [QUEENS' COLL. CAMB.]

235. Sum the series:

(1) $1+2^3x+3^3x^2+\ldots\ldots+n^3x^{n-1}$,

(2) $\dfrac{25}{1^2 . 2^3 . 3^3}+\dfrac{52}{2^2 . 3^3 . 4^3}+\ldots\ldots+\dfrac{5n^3+12n+8}{n^3(n+1)^3(n+2)^3}$.

[EMM. COLL. CAMB.]

236. If $(1+a^3x^4)(1+a^5x^8)(1+a^9x^{16})(1+a^{17}x^{32})\ldots\ldots$

$$=1+A_4x^4+A_8x^8+A_{12}x^{12}+\ldots\ldots$$

prove that $A_{8n+4}=a^3A_{8n}$, and $A_{8n}=a^{2n}A_{4n}$; and find the first ten terms of the expansion. [CORPUS COLL. CAMB.]

237. On a sheet of water there is no current from A to B but a current from B to C; a man rows down stream from A to C in 3 hours, and up stream from C to A in $3\frac{1}{2}$ hours; had there been the same current all the way as from B to C, his journey down stream would have occupied $2\frac{3}{4}$ hours; find the length of time his return journey would have taken under the same circumstances.

238. Prove that the n^{th} convergent to the continued fraction

$$\frac{3}{2+}\,\frac{3}{2+}\,\frac{3}{2+}\ldots\ldots \text{ is } \frac{3^{n+1}+3(-1)^{n+1}}{3^{n+1}-(-1)^{n+1}}.$$

[EMM. COLL. CAMB.]

239. If all the coefficients in the equation

$$x^n+p_1x^{n-1}+p_2x^{n-2}+\ldots\ldots+p_n=f(x)=0,$$

be whole numbers, and if $f(0)$ and $f(1)$ be each odd integers, prove that the equation cannot have a commensurable root.

[LONDON UNIVERSITY.]

240. Shew that the equation

$$\sqrt{ax+\alpha}+\sqrt{bx+\beta}+\sqrt{cx+\gamma}=0$$

reduces to a simple equation if $\sqrt{a}\pm\sqrt{b}\pm\sqrt{c}=0$.

Solve the equation

$$\sqrt{8x^2-12x-39}+\sqrt{2x^2-3x-15}-\sqrt{2x^2-3x+20}=0.$$

241. A bag contains 3 red and 3 green balls, and a person draws out 3 at random. He then drops 3 blue balls into the bag, and again draws out 3 at random. Shew that he may just lay 8 to 3 with advantage to himself against the 3 latter balls being all of different colours. [PEMB. COLL. CAMB.]

242. Find the sum of the fifth powers of the roots of the equation $x^4-7x^2+4x-3=0$. [LONDON UNIVERSITY.]

243. A Geometrical and Harmonical Progression have the same p^{th}, q^{th}, r^{th} terms a, b, c respectively: shew that

$$a(b-c)\log a+b(c-a)\log b+c(a-b)\log c=0.$$

[CHRIST'S COLL. CAMB.]

244. Find four positive numbers such that the sum of the first, third and fourth exceeds the second by 8; the sum of the squares of the first and second exceeds the sum of the squares of the third and fourth by 36; the sum of the products of the first and second, and of the third and fourth is 42; the cube of the first is equal to the sum of the cubes of the second, third, and fourth.

245. If T_n, T_{n+1}, T_{n+2} be 3 consecutive terms of a recurring series connected by the relation $T_{n+2}=aT_{n+1}-bT_n$, prove that

$$\frac{1}{b^n}\{T^2_{n+1}-aT_nT_{n+1}+bT_n^2\}=\text{a constant.}$$

246. Eliminate x, y, z from the equations:

$$\left.\begin{aligned}\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{a},\quad & x^2+y^2+z^2=b^2,\\ x^3+y^3+z^3=c^3,\quad & xyz=d^3.\end{aligned}\right\}$$

[EMM. COLL. CAMB.]

247. Shew that the roots of the equation

$$x^4-px^3+qx^2-rx+\frac{r^2}{p^2}=0$$

are in proportion. Hence solve $x^4-12x^3+47x^2-72x+36=0$.

248. A can hit a target four times in 5 shots; B three times in 4 shots; and C twice in 3 shots. They fire a volley: what is the probability that two shots at least hit? And if two hit what is the probability that it is C who has missed? [ST CATH. COLL. CAMB.]

249. Sum each of the following series to n terms:

(1) $1+0-1+0+7+28+79+\ldots\ldots;$

(2) $-\frac{2.2}{1.2.3.4}+\frac{1.2^2}{2.3.4.5}+\frac{6.2^3}{3.4.5.6}+\frac{13.2^4}{4.5.6.7}+\ldots\ldots;$

(3) $3+x+9x^2+x^3+33x^4+x^5+129x^6+\ldots\ldots$

[SECOND PUBLIC EXAM. OX.]

250. Solve the equations:

(1) $\left.\begin{array}{r} y^2+yz+z^2=ax, \\ z^2+zx+x^2=ay, \\ x^2+xy+y^2=az. \end{array}\right\}$ (2) $\left.\begin{array}{r} x(y+z-x)=a, \\ y(z+x-y)=b, \\ z(x+y-z)=c. \end{array}\right\}$

[PETERHOUSE, CAMB.]

251. If $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{a+b+c}$, and n is an odd integer, shew that

$$\frac{1}{a^n}+\frac{1}{b^n}+\frac{1}{c^n}=\frac{1}{a^n+b^n+c^n}.$$

If $u^6-v^6+5u^2v^2(u^2-v^2)+4uv(1-u^4v^4)=0$, prove that

$$(u^2-v^2)^6=16u^2v^2(1-u^8)(1-v^8).$$ [PEMB. COLL. CAMB.]

252. If $x+y+z=3p$, $yz+zx+xy=3q$, $xyz=r$, prove that

$$(y+z-x)(z+x-y)(x+y-z)=-27p^3+36pq-8r,$$

and $$(y+z-x)^3+(z+x-y)^3+(x+y-z)^3=27p^3-24r.$$

253. Find the factors, linear in x, y, z, of

$$\{a(b+c)x^2+b(c+a)y^2+c(a+b)z^2\}^2-4abc(x^2+y^2+z^2)(ax^2+by^2+cz^2).$$

[CAIUS COLL. CAMB.]

254. Shew that $\left(\frac{x^2+y^2+z^2}{x+y+z}\right)^{x+y+z}>x^xy^yz^z>\left(\frac{x+y+z}{3}\right)^{x+y+z}.$

[ST JOHN'S COLL. CAMB.]

255. By means of the identity $\left\{1-\frac{4x}{(1+x)^2}\right\}^{-\frac{1}{2}}=\frac{1+x}{1-x}$, prove that

$$\sum_{r=1}^{r=n}(-1)^{n-r}\frac{(n+r-1)!}{r!\,(r-1)!\,(n-r)!}=1.$$

[PEMB. COLL. CAMB.]

256. Solve the equations:

(1) $ax+by+z=zx+ay+b=yz+bx+a=0.$

(2) $$\left.\begin{aligned} x+y+z-u &= 12, \\ x^2+y^2-z^2-u^2 &= 6, \\ x^3+y^3-z^3+u^3 &= 218, \\ xy+zu &= 45. \end{aligned}\right\}$$

257. If $p=q$ nearly, and $n>1$, shew that

$$\frac{(n+1)p+(n-1)q}{(n-1)p+(n+1)q}=\left(\frac{p}{q}\right)^{\frac{1}{n}}.$$

If $\frac{p}{q}$ agree with unity as far as the r^{th} decimal place, to how many places will this approximation in general be correct? [MATH. TRIPOS.]

258. A lady bought 54 lbs. of tea and coffee; if she had bought five-sixths of the quantity of tea and four-fifths of the quantity of coffee she would have spent nine-elevenths of what she had actually spent; and if she had bought as much tea as she did coffee and *vice-versâ*, she would have spent 5*s.* more than she did. Tea is more expensive than coffee, and the price of 6 lbs. of coffee exceeds that of 2 lbs. of tea by 5*s.*; find the price of each.

259. If s_n represent the sum of the products of the first n natural numbers taken two at a time, then

$$\frac{2}{3!}+\frac{11}{4!}+\ldots\ldots+\frac{s_{n-1}}{n!}+\ldots\ldots=\frac{11}{24}e.$$

[CAIUS COLL. CAMB.]

260. If $$\frac{P}{pa^2+2qab+rb^2}=\frac{Q}{pac+q(bc-a^2)-rab}=\frac{R}{pc^2-2qca+ra^2},$$

prove that P, p; Q, q; and R, r may be interchanged without altering the equalities. [MATH. TRIPOS.]

261. If $\alpha+\beta+\gamma=0$, shew that

$$\alpha^{n+3}+\beta^{n+3}+\gamma^{n+3}=\alpha\beta\gamma(\alpha^n+\beta^n+\gamma^n)+\frac{1}{2}(\alpha^2+\beta^2+\gamma^2)(\alpha^{n+1}+\beta^{n+1}+\gamma^{n+1}).$$

[CAIUS COLL. CAMB.]

262. If $\alpha, \beta, \gamma, \delta$ be the roots of the equation

$$x^4+px^3+qx^2+rx+s=0,$$

find in terms of the coefficients the value of $\Sigma(\alpha-\beta)^2(\gamma-\delta)^2$.

[LONDON UNIVERSITY.]

263. A farmer bought a certain number of turkeys, geese, and ducks, giving for each bird as many shillings as there were birds of that kind; altogether he bought 23 birds and spent £10. 11*s.*; find the number of each kind that he bought, if geese are cheaper than turkeys and dearer than ducks.

264. Prove that the equation

$$(y+z-8x)^{\frac{1}{3}}+(z+x-8y)^{\frac{1}{3}}+(x+y-8z)^{\frac{1}{3}}=0,$$

is equivalent to the equation

$$x(y-z)^2+y(z-x)^2+z(x-y)^2=0.$$

[St John's Coll. Camb.]

265. If the equation $\frac{a}{x+a}+\frac{b}{x+b}=\frac{c}{x+c}+\frac{d}{x+d}$ have a pair of equal roots, then either one of the quantities a or b is equal to one of the quantities c or d, or else $\frac{1}{a}+\frac{1}{b}=\frac{1}{c}+\frac{1}{d}$. Prove also that the roots are then $-a, -a, 0$; $-b, -b, 0$; or $0, 0, -\frac{2ab}{a+b}$.

[Math. Tripos.]

266. Solve the equations:

(1) $x+y+z=ab$, $x^{-1}+y^{-1}+z^{-1}=a^{-1}b$, $xyz=a^3$.

(2) $ayz+by+cz=bzx+cz+ax=cxy+ax+by=a+b+c$.

[Second Public Exam. Oxford.]

267. Find the simplest form of the expression

$$\frac{\alpha^3}{(\alpha-\beta)(\alpha-\gamma)(\alpha-\delta)(\alpha-\epsilon)}+\frac{\beta^3}{(\beta-\alpha)(\beta-\gamma)(\beta-\delta)(\beta-\epsilon)}+\ldots$$
$$+\frac{\epsilon^3}{(\epsilon-\alpha)(\epsilon-\beta)(\epsilon-\gamma)(\epsilon-\delta)}.$$

[London University.]

268. In a company of Clergymen, Doctors, and Lawyers it is found that the sum of the ages of all present is 2160; their average age is 36; the average age of the Clergymen and Doctors is 39; of the Doctors and Lawyers $32\frac{8}{11}$; of the Clergymen and Lawyers $36\frac{2}{3}$. If each Clergyman had been 1 year, each Lawyer 7 years, and each Doctor 6 years older, their average age would have been greater by 5 years: find the number of each profession present and their average ages.

269. Find the condition, among its coefficients, that the expression

$$a_0x^4+4a_1x^3y+6a_2x^2y^2+4a_3xy^3+a_4y^4$$

should be reducible to the sum of the fourth powers of two linear expressions in x and y. [London University.]

270. Find the real roots of the equations

$$x^2+v^2+w^2=a^2, \quad vw+u(y+z)=bc,$$
$$y^2+w^2+u^2=b^2, \quad wu+v(z+x)=ca,$$
$$z^2+u^2+v^2=c^2, \quad uv+w(x+y)=ab.$$

[MATH. TRIPOS.]

271. It is a rule in Gaelic that no consonant or group of consonants can stand immediately between a strong and a weak vowel; the strong vowels being a, o, u; and the weak vowels e and i. Shew that the whole number of Gaelic words of $n+3$ letters each, which can be formed of n consonants and the vowels aeo is $\dfrac{2\,\lfloor n+3}{n+2}$ where no letter is repeated in the same word. [CAIUS COLL. CAMB.]

272. Shew that if $x^2+y^2=2z^2$, where x, y, z are integers, then

$$2x=r(l^2+2lk-k^2), \quad 2y=r(k^2+2lk-l^2), \quad 2z=r(l^2+k^2)$$

where r, l, and k are integers. [CAIUS COLL. CAMB.]

273. Find the value of $\dfrac{1}{1+}\,\dfrac{1}{1+}\,\dfrac{2}{3+}\,\dfrac{4}{5+}\,\dfrac{6}{7+}\,\ldots\ldots$ to inf.

[CHRIST'S COLL. CAMB.]

274. Sum the series:

(1) $\dfrac{x^2}{2\,.\,3}+\dfrac{2x^3}{3\,.\,4}+\dfrac{3x^4}{4\,.\,5}+\ldots\ldots$ to inf.

(2) $\dfrac{\lfloor 1}{a+1}+\dfrac{\lfloor 2}{(a+1)(a+2)}+\ldots\ldots+\dfrac{\lfloor n}{(a+1)(a+2)\ldots(a+n)}$.

275. Solve the equations:

(1) $2xyz+3=(2x-1)(3y+1)(4z-1)+12$
$$=(2x+1)(3y-1)(4z+1)+80=0.$$

(2) $3ux-2vy=vx+uy=3u^2+2v^2=14;\ xy=10uv.$

276. Shew that
$$\begin{vmatrix} a^2+\lambda & ab & ac & ad \\ ab & b^2+\lambda & bc & bd \\ ac & bc & c^2+\lambda & cd \\ ad & bd & cd & d^2+\lambda \end{vmatrix}$$

is divisible by λ^3 and find the other factor. [CORPUS COLL. CAMB.]

277. If $a, b, c, \ldots$ are the roots of the equation

$$x^n+p_1x^{n-1}+p_2x^{n-2}+\ldots+p_{n-1}x+p_n=0;$$

find the sum of $a^3+b^3+c^3+\ldots$, and shew that

$$\frac{a^2}{b}+\frac{b^2}{a}+\frac{a^2}{c}+\frac{c^2}{a}+\frac{b^2}{c}+\frac{c^2}{b}+\ldots=p_1-\frac{p_{n-1}(p_1^2-2p_2)}{p_n}.$$

[St John's Coll. Camb.]

278. By the expansion of $\dfrac{1+2x}{1-x^3}$, or otherwise, prove that

$$1-3n+\frac{(3n-1)(3n-2)}{1.2}-\frac{(3n-2)(3n-3)(3n-4)}{1.2.3}$$
$$+\frac{(3n-3)(3n-4)(3n-5)(3n-6)}{1.2.3.4}-\&c.=(-1)^n,$$

when n is an integer, and the series stops at the first term that vanishes.

[Math. Tripos.]

279. Two sportsmen A and B went out shooting and brought home 10 birds. The sum of the squares of the number of shots was 2880, and the product of the numbers of shots fired by each was 48 times the product of the numbers of birds killed by each. If A had fired as often as B and B as often as A, then B would have killed 5 more birds than A: find the number of birds killed by each.

280. Prove that $8(a^3+b^3+c^3)^2>9(a^2+bc)(b^2+ca)(c^2+ab)$.

[Pemb. Coll. Camb.]

281. Shew that the n^{th} convergent to

$$\frac{2}{3-}\,\frac{4}{4-}\,\frac{6}{5-}\,\ldots \text{ is } 2-\frac{2^{n+1}}{\Sigma_0^n 2^r(n-r)!}.$$

What is the limit of this when n is infinite? [King's Coll. Camb.]

282. If $\dfrac{p_n}{q_n}$ is the n^{th} convergent to the continued fraction

$$\frac{1}{a+}\,\frac{1}{b+}\,\frac{1}{c+}\,\frac{1}{a+}\,\frac{1}{b+}\,\frac{1}{c+}\,\ldots$$

shew that $p_{3n+3}=bp_{3n}+(bc+1)q_{3n}$. [Queens' Coll. Camb.]

283. Out of n straight lines whose lengths are 1, 2, 3, ... n inches respectively, the number of ways in which four may be chosen which will form a quadrilateral in which a circle may be inscribed is

$$\frac{1}{48}\{2n(n-2)(2n-5)-3+3(-1)^n\}.$$

[Math. Tripos.]

284. If u_2, u_3 are respectively the arithmetic means of the squares and cubes of all numbers less than n and prime to it, prove that $n^3-6nu_2+4u_3=0$, unity being counted as a prime.

[ST JOHN'S COLL. CAMB.]

285. If n is of the form $6m-1$ shew that $(y-z)^n+(z-x)^n+(x-y)^n$ is divisible by $x^2+y^2+z^2-yz-zx-xy$; and if n is of the form $6m+1$, shew that it is divisible by

$$(x^2+y^2+z^2-yz-zx-xy)^2.$$

286. If S is the sum of the m^{th} powers, P the sum of the products m together of the n quantities $a_1, a_2, a_3, \ldots a_n$, shew that

$$\underline{|n-1} \, . \, S > \underline{|n-m} \, . \, \underline{|m} \, . \, P.$$

[CAIUS COLL. CAMB.]

287. Prove that if the equations

$$x^3+qx-r=0 \text{ and } rx^3-2q^2x^2-5qrx-2q^3-r^2=0$$

have a common root, the first equation will have a pair of equal roots; and if each of these is a, find all the roots of the second equation.

[INDIA CIVIL SERVICE.]

288. If $\quad x\sqrt{2a^2-3x^2}+y\sqrt{2a^2-3y^2}+z\sqrt{2a^2-3z^2}=0,$
where a^2 stands for $x^2+y^2+z^2$, prove that

$$(x+y+z)(-x+y+z)(x-y+z)(x+y-z)=0.$$

[TRIN. COLL. CAMB.]

289. Find the values of $x_1, x_2, \ldots x_n$ which satisfy the following system of simultaneous equations:

$$\frac{x_1}{a_1-b_1}+\frac{x_2}{a_1-b_2}+\ldots+\frac{x_n}{a_1-b_n}=1,$$

$$\frac{x_1}{a_2-b_1}+\frac{x_2}{a_2-b_2}+\ldots+\frac{x_n}{a_2-b_n}=1,$$

....

$$\frac{x_1}{a_n-b_1}+\frac{x_2}{a_n-b_2}+\ldots+\frac{x_n}{a_n-b_n}=1.$$

[LONDON UNIVERSITY.]

290. Shew that $\begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ zx-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix} = \begin{vmatrix} r^2 & u^2 & u^2 \\ u^2 & r^2 & u^2 \\ u^2 & u^2 & r^2 \end{vmatrix},$

where $r^2=x^2+y^2+z^2$, and $u^2=yz+zx+xy$.

[TRIN. COLL. CAMB.]

291. A piece of work was done by A, B, C; at first A worked alone, but after some days was joined by B, and these two after some days were joined by C. The whole work could have been done by B and C, if they had each worked twice the number of days that they actually did. The work could also have been completed without B's help if A had worked two-thirds and C four times the number of days they actually did; or if A and B had worked together for 40 days without C; or if all three had worked together for the time that B had worked. The number of days that elapsed before B began to work was to the number that elapsed before C began to work as 3 to 5: find the number of days that each man worked.

292. Shew that if S_r is the sum of the products r together of

$$1,\ x,\ x^2,\ x^3,\ \dots\ x^{n-1},$$

then

$$S_{n-r} = S_r \,.\, x^{\frac{1}{2}(n-1)(n-2r)}.$$

[ST JOHN'S COLL. CAMB.]

293. If a, b, c are positive and the sum of any two greater than the third, prove that

$$\left(1+\frac{b-c}{a}\right)^a \left(1+\frac{c-a}{b}\right)^b \left(1+\frac{a-b}{c}\right)^c < 1.$$

[ST JOHN'S COLL. CAMB.]

294. Resolve into factors

$$(a+b+c)(b+c-a)(c+a-b)(a+b-c)(a^2+b^2+c^2) - 8a^2b^2c^2.$$

Prove that

$$4\{\alpha^4+\beta^4+\gamma^4+(\alpha+\beta+\gamma)^4\} = (\beta+\gamma)^4+(\gamma+\alpha)^4+(\alpha+\beta)^4$$
$$+6(\beta+\gamma)^2(\gamma+\alpha)^2+6(\gamma+\alpha)^2(\alpha+\beta)^2+6(\alpha+\beta)^2(\beta+\gamma)^2.$$

[JESUS COLL. CAMB.]

295. Prove that the sum of the homogeneous products of r dimensions of the numbers 1, 2, 3, ... n, and their powers is

$$\frac{(-1)^{n-1}}{|\underline{n-1}}\left\{1^{n+r-1} - \frac{n-1}{1}\,.\,2^{n+r-1} + \frac{(n-1)(n-2)}{1\,.\,2}\,.\,3^{n+r-1} - \dots \text{ to } n \text{ terms}\right\}.$$

[EMM. COLL. CAMB.]

296. Prove that, if n be a positive integer,

$$1 - 3n + \frac{3n(3n-3)}{1\,.\,2} - \frac{3n(3n-4)(3n-5)}{1\,.\,2\,.\,3} + \dots = 2(-1)^n.$$

[OXFORD MODS.]

297. If $x(2a-y) = y(2a-z) = z(2a-u) = u(2a-x) = b^2$, shew that $x=y=z=u$ unless $b^2 = 2a^2$, and that if this condition is satisfied the equations are not independent. [MATH. TRIPOS.]

298. Shew that if a, b, c are positive and unequal, the equations

$$ax+yz+z=0, \quad zx+by+z=0, \quad yz+zx+c=0,$$

give three distinct triads of real values for x, y, z; and the ratio of the products of the three values of x and y is $b(b-c):a(c-a)$.

[OXFORD MODS.]

299. If
$$A=ax-by-cz, \quad D=bz+cy,$$
$$B=by-cz-ax, \quad E=cx+az,$$
$$C=cz-ax-by, \quad F=ay+bx,$$

prove that $ABC-AD^2-BE^2-CF^2+2DEF$

$$=(a^2+b^2+c^2)(ax+by+cz)(x^2+y^2+z^2).$$

[SECOND PUBLIC EXAM. OXFORD.]

300. A certain student found it necessary to decipher an old manuscript. During previous experiences of the same kind he had observed that the number of words he could read daily varied jointly as the number of miles he walked and the number of hours he worked during the day. He therefore gradually increased the amount of daily exercise and daily work at the rate of 1 mile and 1 hour per day respectively, beginning the first day with his usual quantity. He found that the manuscript contained 232000 words, that he counted 12000 on the first day, and 72000 on the last day; and that by the end of half the time he had counted 62000 words: find his usual amount of daily exercise and work.